

Entropy for Color Superconductivity in Quark Matter

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Abstract

We study a model for color superconductivity with both three colors and massless flavors including quark pairing. By using the Hamiltonian in the color-flavor basis we can calculate the quantum entropy. From this we are able to further investigate the phases of the color superconductor, for which we find a rather sharp transition to color superconductivity above a chemical potential around 290 MeV.

1 Introduction

At high quark chemical potentials and low temperatures the internal structure of hadronic matter has been conjectured to dissolve into a degenerate system of quarks. Such material consisting of very cold dense quarks might exist in the interior of compact stellar objects. However, due to the difficulties of performing lattice simulations with high chemical potentials, it is still not possible to simulate the physics of these phases by using the usual lattice gauge computations. Nevertheless, a nonperturbative analysis at finite baryon density has been quite recently carried out on the lattice by using

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the Nambu-Jona-Lasinio model. The degenerate quarks near to the Fermi surfaces are generally expected to interact according to quantum chromodynamics (QCD) so that they can build up Cooper pairs. This process may lead to superconducting quark matter [1].

Before we present the model for the quantum entropy S in color superconducting quark matter, one might well ask why S as a physical quantity is at all significant. There are a number of physical systems, which we have previously discussed in [2–4], for which one has to reconsider the meaning the third law of thermodynamics in its origin form. We have found that the entropy remains finite even at absolute zero. However, if one were to take this fact into consideration, some aspects of the thermodynamical formulation would become more complicated. Our objective here is to interpret the reason for the finiteness of this entropy at $T = 0$ in the quark matter in relation to the quantum correlations in the ground state. How the presence of this finite quantum entropy term would affect the actual thermodynamics at finite temperatures will be discussed elsewhere. The effects of including S in the hadronic equation of state at low temperatures we have already studied in special cases [4]. To our knowledge, the works of Elze [6] have provided a start for addressing the question about the origin of the *entropy puzzle* in the high-energy collisions. These works established a theoretical framework for discussing how two hadronic scattering initial states undergo hard collisions in quantum mechanically pure initial states. This situation can result in a high-multiplicity event corresponding to a highly impure thermal density matrix on the partonic level before hadronization. According to these works [6] the entropy is an unambiguous characteristic property of the quantum nature of the system. The entropy production is clearly due to the *environmentally induced quantum decoherence* in the observable subsystem. Therefore, we consider that there is no obvious theoretical reason to consider finite entropy for the interpretation of the particle multiplicity, while then explicitly setting its value to zero in other comparable cases. We have also used the prescription of von Neumann for the entropy, which make use of the eigenvalues of the reduced density matrices. Thus we are able to give a first quantitative evaluation for the quarks' entropy inside the hadrons [2–4] under the condition that in the singlet state the quark groundstates are maximally mixed. In a previous work [5] we have given a general evaluation of the entropy for the condensate in quark color superconductivity using a nonrelativistic model based on the BCS theory .

In this work we start with an ultrarelativistic model Hamiltonian for

quarks with three colors and flavors which was proposed a few years ago [7, 8]. In this framework we shall apply our previous calculations for the quantum ground state entropy [2–4] on these quarks under at large values of the quark chemical potential. As mentioned above, the quantum entropy means that entropy which arises from quantum correlations from the quantum fluctuations. These entities differ from the usual thermal fluctuations of a statistical system in that they can also exists at zero temperature. In the above mentioned model with three colors and flavors a total of nine mixed quark states are present. For such mixed states in the pairs of two colored quarks the quantum entropy can be expected to be temperature independent and equal to just $9 \ln 4$ in maximally mixed states [5].

2 Color Superconductor Model

We now look more explicitly at the effects on the ground state in our model with three colors and three flavors for the structure of the superconductivity. In this part we shall calculate the quantum entropy from the gap equations for the color superconducting state. For massless quarks the form for the Hamiltonian already has been written down for the three color-flavor superconductivity [7] arising from quark-pairs at low temperatures and high quark chemical potentials. It takes the following form:

$$\begin{aligned}
H = & \sum_{\rho, k > \mu} (k - \mu) a_{\rho}^{\dagger}(\mathbf{k}) a_{\rho}(\mathbf{k}) + \sum_{\rho, k < \mu} (\mu - k) a_{\rho}^{\dagger}(\mathbf{k}) a_{\rho}(\mathbf{k}) + \sum_{\rho, \mathbf{k}} (k + \mu) b_{\rho}^{\dagger}(\mathbf{k}) b_{\rho}(\mathbf{k}) \\
& + \frac{1}{2} \sum_{\rho, \mathbf{p}} F(p)^2 Q_{\rho} e^{-i\phi(\mathbf{p})} (a_{\rho}(\mathbf{p}) a_{\rho}(-\mathbf{p}) + b_{\rho}^{\dagger}(\mathbf{p}) b_{\rho}^{\dagger}(-\mathbf{p})) \\
& + \frac{1}{2} \sum_{\rho, \mathbf{p}} F(p)^2 Q_{\rho} e^{i\phi(\mathbf{p})} (a_{\rho}^{\dagger}(\mathbf{p}) a_{\rho}^{\dagger}(-\mathbf{p}) + b_{\rho}(\mathbf{p}) b_{\rho}(-\mathbf{p})). \tag{1}
\end{aligned}$$

$F(p)^2$ is the form factor containing the cut-off Λ . Q_{ρ} stands for the diagonalized form for the gap parameters, for which $\rho = 1$ yields the color-flavor singlet gap-parameter Δ_1 and $\rho = 2, \dots, 9$ result in the color-flavor gap $\pm \Delta_8$. The first line of the Hamiltonian represents only the non-interacting parts, while the second and third lines thereof are the complex conjugate terms of the interactions with opposite momenta. a_{ρ}^{\dagger} and a_{ρ} together with b_{ρ}^{\dagger} and b_{ρ} are the creation and annihilation operators of the particle and antiparticle

states, respectively. The index ρ , as given above, stands for both the color and flavor degrees of freedom.

For our present purpose we can treat ρ in the same way as we had previously taken only the color degrees of freedom since the flavors now provide an exact symmetry in the limit of massless quarks. We take μ as the quark chemical potential, for which all the momenta up to $\mu = p_F$ have all the particle and antiparticle states completely occupied in the groundstate.

A proper parameterization for the annihilation and creation operators, respectively, had been already suggested [7] as follows:

$$y_\rho(\mathbf{k}) = \cos[\theta_\rho^y(\mathbf{k})]a_\rho(\mathbf{k}) + \sin[\theta_\rho^y(\mathbf{k})]e^{i\xi_\rho^y(\mathbf{k})}a_\rho^\dagger(-\mathbf{k}) \quad (2)$$

$$z_\rho(\mathbf{k}) = \cos[\theta_\rho^z(\mathbf{k})]b_\rho(\mathbf{k}) + \sin[\theta_\rho^z(\mathbf{k})]e^{i\xi_\rho^z(\mathbf{k})}b_\rho^\dagger(-\mathbf{k}) \quad (3)$$

$$y_\rho^\dagger(\mathbf{k}) = \cos[\theta_\rho^y(\mathbf{k})]a_\rho^\dagger(\mathbf{k}) + \sin[\theta_\rho^y(\mathbf{k})]e^{-i\xi_\rho^y(\mathbf{k})}a_\rho(-\mathbf{k}) \quad (4)$$

$$z_\rho^\dagger(\mathbf{k}) = \cos[\theta_\rho^z(\mathbf{k})]b_\rho^\dagger(\mathbf{k}) + \sin[\theta_\rho^z(\mathbf{k})]e^{-i\xi_\rho^z(\mathbf{k})}b_\rho(-\mathbf{k}) \quad (5)$$

Therefrom the following definitions are given:

$$\theta_\rho^y(\mathbf{k}) = \frac{1}{2} \arccos \left(\frac{|k - \mu|}{\sqrt{(k - \mu)^2 + F(k)^4 Q_\rho^2}} \right) \quad (6)$$

$$\xi_\rho^y(\mathbf{k}) = \phi(\mathbf{k}) + \pi \quad (7)$$

$$\theta_\rho^z(\mathbf{k}) = \frac{1}{2} \arccos \left(\frac{|k + \mu|}{\sqrt{(k + \mu)^2 + F(k)^4 Q_\rho^2}} \right) \quad (8)$$

$$\xi_\rho^z(\mathbf{k}) = -\phi(\mathbf{k}) \quad (9)$$

We may compare these complex expressions with the usual forms for the Bogoliubov transformations, from which we can write

$$u_\rho(\mathbf{k}) \equiv \cos[\theta_\rho(\mathbf{k})], \quad (10)$$

$$v_\rho(\mathbf{k}) \equiv \sin[\theta_\rho(\mathbf{k})]e^{i\xi_\rho(\mathbf{k})}, \quad (11)$$

Obviously, we get

$$u_\rho^*(\mathbf{k})u_\rho(\mathbf{k}) + v_\rho^*(\mathbf{k})v_\rho(\mathbf{k}) = 1, \quad (12)$$

which shows the canonical nature of these transformations.

After we have carried out these canonical transformations, the form of the Hamiltonian for noninteracting quasiquarks takes on the quadratic canonical structure:

$$H = \sum_{\mathbf{k}, \rho} \left[\left((k - \mu)^2 + F(k)^4 Q_\rho^2 \right)^{1/2} y_\rho^\dagger(\mathbf{k}) y_\rho(\mathbf{k}) + \left((k + \mu)^2 + F(k)^4 Q_\rho^2 \right)^{1/2} z_\rho^\dagger(\mathbf{k}) z_\rho(\mathbf{k}) \right] \quad (13)$$

We now write out the following definitions:

$$\Upsilon_\rho^y(\mathbf{k}) \equiv \frac{u_\rho^{y*}(\mathbf{k}) u_\rho^y(\mathbf{k})}{v_\rho^{y*}(\mathbf{k}) v_\rho^y(\mathbf{k})} = \frac{\sqrt{(k - \mu)^2 + F(k)^4 Q_\rho^2} + |k - \mu|}{\sqrt{(k - \mu)^2 + F(k)^4 Q_\rho^2} - |k - \mu|} \quad (14)$$

$$\Upsilon_\rho^z(\mathbf{k}) \equiv \frac{u_\rho^{z*}(\mathbf{k}) u_\rho^z(\mathbf{k})}{v_\rho^{z*}(\mathbf{k}) v_\rho^z(\mathbf{k})} = \frac{\sqrt{(k + \mu)^2 + F(k)^4 Q_\rho^2} + |k + \mu|}{\sqrt{(k + \mu)^2 + F(k)^4 Q_\rho^2} - |k + \mu|} \quad (15)$$

For $k \neq 0$ we write the quantum entropy of entanglement for this color superconducting model in the following form:

$$S_{CSM} = g \sum_{\rho=1}^9 \left\{ \frac{\ln \Upsilon_\rho^y(\mathbf{k})}{\Upsilon_\rho^y(\mathbf{k}) + 1} + \ln \left(1 + \frac{1}{\Upsilon_\rho^y(\mathbf{k})} \right) + \frac{\ln \Upsilon_\rho^z(\mathbf{k})}{\Upsilon_\rho^z(\mathbf{k}) + 1} + \ln \left(1 + \frac{1}{\Upsilon_\rho^z(\mathbf{k})} \right) \right\} \quad (16)$$

We first must set up the equations for the gaps Δ_1 and Δ_8 , which have been previously studied by Alford, Rajagopal and Wilczek [7]. Before we write down the complete gap equations, we briefly discuss the two color flavor superconducting model known as the 2SC Model. In structure it is very similar to the simple nonrelativistic BCS model with the addition of the antiparticle contribution. A not too great generalization of this type of model leads to a two flavor and three color model [7], which is the prior step to the above model. The 2SC phase is such that the diquarks condense while the chiral symmetry is restored. It has a simple equation for the gap Δ similar to the above BCS Model. We write the gap equation in the form [8]

$$1 = \frac{2G}{V} \sum_p \left\{ \frac{1}{\sqrt{(p - \mu)^2 + \Delta^2}} + \frac{1}{\sqrt{(p + \mu)^2 + \Delta^2}} \right\}. \quad (17)$$

We can convert the sum over all the momenta p into an integral $\mathcal{I}[\Delta]$ so that

$$\mathcal{I}[\Delta] = \frac{G}{\pi^2} \int_0^\Lambda k^2 dk \left(\frac{1}{\sqrt{(k-\mu)^2 + \Delta^2}} + \frac{1}{\sqrt{(k+\mu)^2 + \Delta^2}} \right) \quad (18)$$

This form of the gap equation

$$1 = \mathcal{I}[\Delta] \quad (19)$$

can be evaluated and substituted in the above equation for the entropy. However, it does not properly reflect the fully extended color-flavor symmetry of our above model Hamiltonian. Nevertheless, we can use this simpler equation with the constituent quark mass M replacing the gap parameter at vanishing chemical potential in order to determine the coupling G . The coupled integral equations for the gaps represent the singlet and octet decomposition of this extended symmetry [7]. In our evaluation of the equations for the gaps Δ_1 and Δ_8 we use the step-function cut-off with $\Lambda = 800$ MeV. Thus we write the two gap equations [8] in the following form:

$$\Delta_1 = -2\Delta_8 \mathcal{I}[\Delta_8] \quad (20)$$

$$\Delta_8 = -\Delta_1 (1 + \mathcal{I}[\Delta_1]) / 4 \quad (21)$$

We use these gap equations for Δ_1 and Δ_8 in order to obtain $\Upsilon_\rho^y(\mathbf{k})$ and $\Upsilon_\rho^z(\mathbf{k})$ with $F(k) = 1$ for $k < \Lambda$ and zero above. These quantities are substituted into the above equation for the quantum entropy S_{CSM} . The results for $9 \ln 4 - S_{CSM}$ are shown in Fig. 1 for different values of the quark chemical potential μ . We can see the effects of the gaps Δ_1 and Δ_8 on the quantum entropy. Below a critical value of the chemical potential around 290 MeV the gaps vanish so that the spread of S_{CSM} also vanishes. We see this in a very narrow line that extends downwards to zero at values of μ under 290 MeV.

3 Results and discussion

Now we discuss the results from the computation of the quantum entropy S_{CSM} derived in the equation 16. Figure 1 shows the difference between the entropy from the quark pairing in color superconductors Eq. 16 and

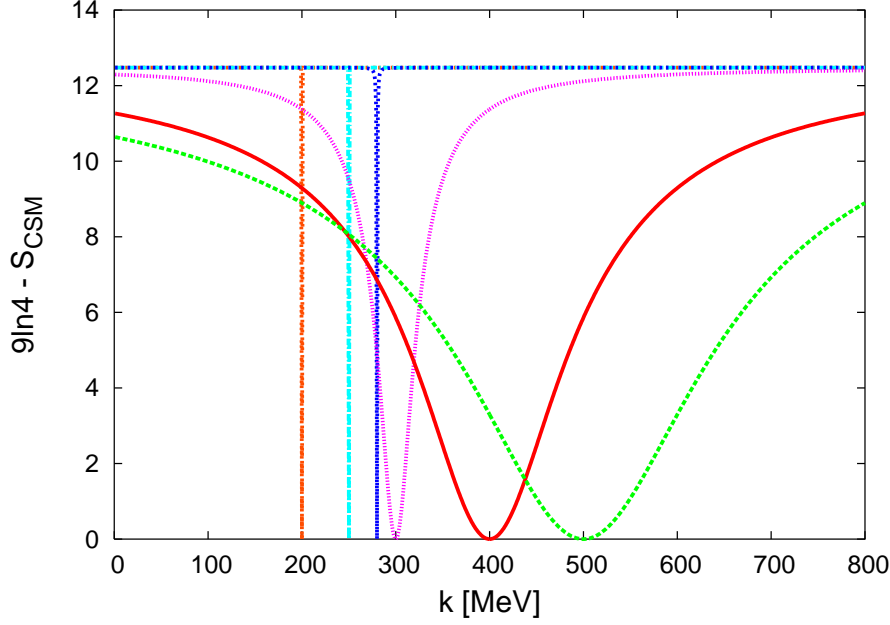


Fig. 1: *The momentum dependence of the difference between the maximum entropy in the ground state $9 \ln 4$ and the entropy from the excitation of color superconductors is shown for different values of the chemical potential μ indicated by the zero-point on the graph.*

the maximum value of the groundstate entropy for nine states given by $9 \ln 4$. The dependence of $9 \ln 4 - S_{CSM}$ upon the momenta k is plotted for different values of the chemical potential μ . This difference has a zero-point when the value of the momentum $k = \mu$. In this figure we notice for the zero-point values above $\mu = 290$ that there is always a finite spread in the curve around the zero-point. From this fact we can extract at the halfheight value the fullwidth Γ . By means of a direct comparison with the Gaussian distribution we take the dispersion to be the standard deviation σ . In this case we have simply the fullwidth $\Gamma = 2\sqrt{2 \ln 2} \sigma$ at half maximum. Then we can compute the dispersion σ from the distributions given in the next figure 2 as a function of the corresponding μ value. In this model with three massless flavors taken together with the three colors from the usual $SU(3)_c$ we are able to analyze the dependence of the gaps Δ_1 and Δ_8 on the quark chemical potential μ using the physical quantity S_{CSM} .

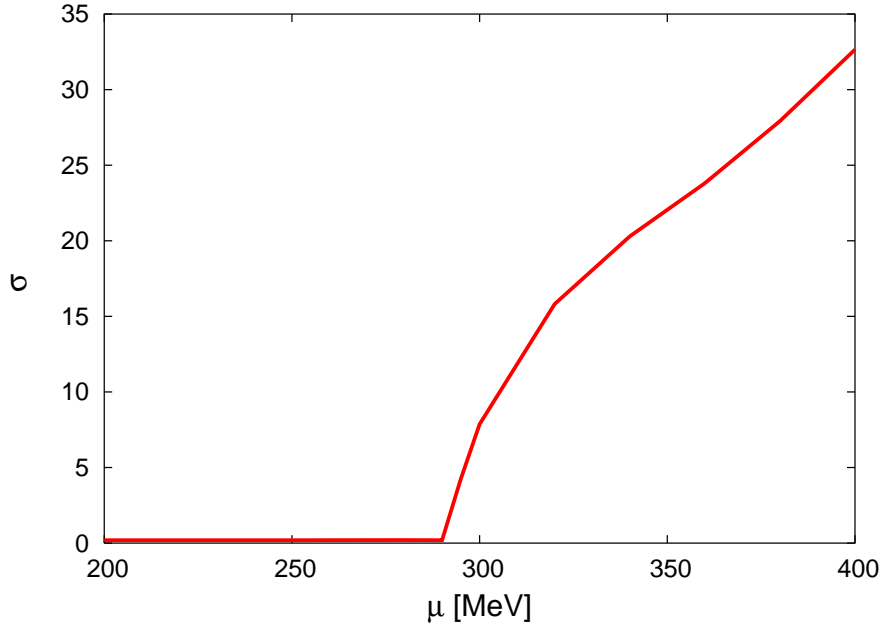


Fig. 2: *The dispersion σ taken from the distributions given in Fig. 1 plotted against the corresponding μ value. $\sigma \neq 0$ for quark chemical potential $\mu > 290$ MeV.*

4 Conclusion

Finally we can conclude that the quantum entropy is a good indicator of the phase transition arising from the presence of the two gaps Δ_1 and Δ_8 . As we have seen in the figure 2, the dispersion σ becomes finite at a chemical potential of about 290 MeV, which is very near the previously calculated value of around 300 MeV for the gaps appearing in the CFL phase [7, 8]. Further discussion of these models and their properties relating to the phenomena of color superconductivity has appeared quite recently [10]. In our present work we have shown that by using the entropy S_{CSM} we have an additional quantity which can be computed to show the transition to color superconductivity. Its deviation from the pairing value is due to the two gaps. Furthermore, the dispersion σ relates directly to the correlations, which, in this case, these correlations are between the quarks within the pairs, which is similar to the diquark structure. These ideas could lead to further investigations relating to the expected correlations between the diquarks as well as, perhaps, find future applications in the theoretical

studies of the recently experimentally discovered pentaquark states [12]. These states can be interpreted [11] as a bound state of four quarks and an antiquark, which consist of two highly correlated quark pairs.

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References

- [1] B. C. Barrois, *Nucl. Phys.*, B129:390, 1977.
- [2] D. E. Miller, *Entropy for $SU(3)_c$ quark states*, Eur. Phys. J. C, 2004, hep-ph/0306302
- [3] D. E. Miller and A. Tawfik, *Entropy for colored quark states at finite temperature*, hep-ph/0308192;
S. Hamieh and A. Tawfik, *Finite temperature quantum correlations in $SU(2)_c$ quark states and quantum spin models*, hep-ph/0404246
- [4] D. E. Miller and A. Tawfik, *The effects of colored quark entropy on the bag pressure*, J. Phys. G30:731, 2004;
The effects of quantum entropy on the bag constant hep-ph/0309139
- [5] D. E. Miller and A. Tawfik, *Entanglement in Condensates involving Strong Interactions*, hep-ph/0312368
- [6] H.-T. Elze, Nucl. Phys. **B** 436: 213, 1995;
Nucl. Phys. Proc. Suppl. **39BC**: 169, 1995;
Phys. Lett. **B** 369: 295, 1996
- [7] M. Alford, K. Rajagopal and F. Wilczek, *Phys. Lett.*, 422B:247, 1998;
Nucl. Phys., B537:443, 1999.
- [8] K. Rajagopal and F. Wilczek, *The condensed matter physics of QCD*, Handbook of QCD: At the Frontier of Particle Physics, M. Shifman, ed., (World Scientific) 2000.
- [9] M. Alford, J. Berges and K. Rajagopal, *Nucl. Phys.*, B558:219, 1999.
- [10] J. A. Bowers, *Color superconducting phases of cold dense quark matter*, arXiv:hep-ph/0305301.
- [11] R. Jaffe and F. Wilczek, *Phys. Rev. Lett.*, 91:232003, 2003.
- [12] T. Nakano et al., *Phys. Rev. Lett.*, 91:012002, 2003;
C. Alt et al., *Phys. Rev. Lett.*, 92:042003, 2004.